

# OGAP

## Proportionality Framework

(Draft June 2011)

Depending upon the strength of proportional reasoning students may move back and forth between using proportional, transitional, and non-proportional strategies as they interact with different problem structures and problem situations. (Cramer, Post & Currier, 1993; Karplus, Pulos & Stage, 1983; VMP OGAP, 2006 & 2007)

### PROBLEM STRUCTURES

<b><u>PROBLEM SITUATIONS</u></b>	<b>Problem Types</b>	<b>Multiplicative Relationships</b>	<b>Ratio Relationships</b>
Ratios	Ratio	Both integral	Part: whole
Rates (Density)	Unit Rate	Both non-integral	Part: part
Rates (D=RT)	Ratio and rate comparisons	Some non-integral	
Rates (Buy/consume)	Missing value		<b>Ratio Referent</b>
Measure conversions	Scale factor	<b>Numbers</b>	Implied
Similarity	Qualitative	All integers	Explicit
Scale	Non-proportional	All non-integers	
Probability		Both integer and non-integer	<b>Representations</b>
Percent	<b>Internal Structures</b>		Coordinate Graph
Linear equations and relationships	Parallel	<b>Language</b>	Table
Slope	Non-parallel	Natural (e.g., every)	Model
Frequency distributions		Mathematical (e.g., per)	Expression
			Equation

*OGAP references are found at [http://www.margepetit.com/petit\\_pdfs/OGAPReferences3.pdf](http://www.margepetit.com/petit_pdfs/OGAPReferences3.pdf).*

### ABOUT OGAP FRAMEWORKS

OGAP Frameworks are based on mathematics education research on how students learn specific mathematics concepts, errors students make, and pre-conceptions or misconceptions that may interfere with learning new concepts or solving related problems.

There are three major elements to an OGAP Framework that should be considered when analyzing student work or making instructional decisions:

- 1) problem situations
- 2) problem structures
- 3) evidence in student work

This page identifies problem situations and problem structures for ratio and proportion problems. Pages 2 and 3 are tools to help teachers classify evidence in student work, including classroom discussions, and make instructional decisions. Page 4 has some samples of different problem situations and problem types.

For students to become strong in their application of ratios, rates, and proportional relationships they must interact with a range of problem situations and problem structures. Beginning in grade 4 the CCSSM engages students in multiplicative change problems which set the foundation for proportional thinking. In grade 6 students focus on understanding ratios and unit rates and applying these concepts in problems involving unit pricing, constant rate of speed, and measure conversions. In grade 7 this work is extended to proportional relationships and solving multi-step proportion problems, percent problems, scale problems, and developing probability. In grade 8 this work is extended to understanding the connections between proportional relationships, lines, and linear equations including understanding of similarity.

Consistent with the CCSSM the OGAP Frameworks on pages 2 and 3 shows a progression from using models, building up and down, and ratio tables to efficient and generalized strategies identified in Proportional Strategies in the OGAP Proportionality Framework.

As students interact with new concepts, new problem situations, new structures, and more complex problem solving situations they may move back and forth between proportional, transitional, and non-proportional strategies. This is important evidence to use for instructional decision making. For example, a student may consistently solve problems involving unit rate. However, when given a density comparison problem students may revert to a non-proportional strategy.

**Finds and applies a unit rate**

Bob's shower uses 18 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 13 minutes? Show your work.

$$\frac{18 \text{ gal}}{3 \text{ min}} \times 13 = 78 \text{ gal}$$

**Applies multiplicative relationship**

Bob's shower uses 18 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 13 minutes? Show your work.

$$18 \text{ gallons} \times \frac{4\frac{1}{3}}{3} = 78$$

If Bob takes a 13 minute shower he will use 78 gallons of water.

**Uses  $y=kx$  (either symbolic or graphic representation)**

Bob's shower uses 12 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 13 minutes? Show your work.

$$y = kx$$

$$y = 4 \text{ gallons/minute} \times 13 \text{ minutes}$$

$$y = 52 \text{ gallons}$$

**Uses ratio table**

Bob's shower uses 18 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 12 minutes? Show your work.

Time (min)	3	6	12
Gallons	18	36	72

**Builds up or down**

Bob's shower uses 18 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 12 minutes? Show your work.

$$18 \text{ gallons} = 3 \text{ min}$$

$$36 \text{ gallons} = 6 \text{ min}$$

$$54 \text{ gallons} = 9 \text{ min}$$

$$72 \text{ gallon} = 12 \text{ min}$$

**Uses model**

Carrie is packing apples for an orchard's mail order business. It takes 3 boxes to pack 2 bushels of apples. How many boxes will she need to pack 7 bushels of apples?

□ = bushel    ■ = box



**Compares simplified fractions, rates, and ratios**

The dimensions of 4 rectangles are given below. Which 2 rectangles are similar? Show your work.

Explain your reasoning.

A. 2" x 8"

B. 4" x 10"

C. 6" x 12"

D. 6" x 15"

A length/width =  $\frac{2}{8} = \frac{1}{4}$

B length/width =  $\frac{4}{10} = \frac{2}{5}$

C  $\frac{6}{12} = \frac{1}{2}$

D  $\frac{6}{15} = \frac{2}{5}$

**Sets up a proportion and uses cross products**

Bob's shower uses 14 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 8 minutes? Show your work.

$$\frac{3 \text{ minutes}}{14 \text{ gallons}} = \frac{8 \text{ minutes}}{x}$$

$$14 \text{ gallons} \times 8 \text{ min} = 112$$

$$112 \div 3 \text{ minutes} = 37.3 \text{ gallons}$$

**Applies the correct ratio referent in a ratio problem**

There are red and blue marbles in a bag. The ratio of red to blue marbles is 1:2. Sue opened the bag and found 12 red marbles. How many marbles are in the bag altogether?

Red 1, Blue 2

Red 12, Blue 24

36 marbles in the bag

**Proportional strategy with an error**

**Multiplicative relationship error**

Paul's dog eats 20 pounds of food in 30 days. How long will it take Paul's dog to eat a 45 pound bag of dog food?

$$\frac{20 \text{ lb}}{30 \text{ days}} = \frac{45 \text{ lb}}{x \text{ days}}$$

$$65 = 30 \times 2.15$$

$$45 = 20 \times 2.15$$

**Sets up proportion, but misapplies cross products**

$$\frac{4}{12} \times \frac{6}{x}$$

$$4 \times 72 = 288$$

**Finds equivalent fractions/ratios with an error.**

Other

Depending upon the strength of proportional reasoning students may move back and forth between proportional, transitional, non-proportional strategies as they interact with different problem situations and problem structures. (Cramer, Post & Currier, 1993; Karplus, Pulos & Stage, 1983; VMP OGAP, 2006 & 2007)

## Uses additive reasoning

Bob's shower uses 14 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 8 minutes? Show your work.

$$\begin{array}{r} +5 \\ \hline 3 \text{ min} \quad 8 \text{ min} \\ +5 \\ \hline 14 \text{ gal} \quad 19 \text{ gal} \end{array}$$

## Guesses or uses random operations

Carrie is packing apples for an orchard's mail order business. It takes 3 boxes to pack 2 bushels of apples. How many boxes will she need to pack 7 bushels of apples?

$$\begin{array}{r} 7 \\ \times 3 \\ \hline 21 \end{array}$$

7/4

$$\begin{array}{r} \times 2 \\ \hline 14 \\ \times 4 \\ \hline 56 \end{array}$$

6, 12, 18, 24, 30, 36

## Misinterpret vocabulary and related concept

*I don't know what this means.*

M & Ms  
The ratio of red M&Ms to the rest of the M&Ms in a bag is 1:2.

Sue opened the bag and found 12 red M&Ms.

How many M&Ms were in the bag altogether? Explain your thinking

key:  $\bullet$  = Red  
○ = rest

14 M&Ms all together.

## Uses whole number reasoning

Town A	Town B
60 square miles	40 square miles
480 raccoons	380 raccoons

Karl says that Town A has more raccoons per square mile. Josh says Town B has more raccoons per square mile. Who is right? Justify your answer.

answei.

(A)  $\frac{480}{60} = 8$

(B)  $\frac{380}{40} = 9.5$

Karl is right there are more cause of 60 and 480 and all together is 1080 and it just even more cause of 60 and 480 while 40 and 380 are small to it. I think equals 426 so it's still small.

## Solves a non-proportional problem using proportions

Kim and Bob were running equally fast around a track. Kim started first. When she had run 9 laps, Bob had run 3 laps. When Bob had run 15 laps, how many laps had Kim run?

Kim =  $\frac{9 \text{ laps}}{3 \text{ laps}} = \frac{45 \text{ laps}}{15 \text{ laps}}$

Bob =  $\frac{3 \text{ laps}}{15 \text{ laps}}$

Kim had run 45 laps.

## Uses incorrect ratio referent

Other

## Underlying Issues/Errors

### Uses additive strategy to find multiplicative relationship

Town A	Town B
60 square miles	40 square miles
480 raccoons	380 raccoons

Karl says that Town A has more raccoons per square mile. Josh says Town B has more raccoons per square mile. Who is right? Justify your answer.

Town A has 8 raccoons 60 per square mile  
Town B has around 9 raccoons per square mile

$$\begin{array}{r} 60 \text{ per } 3 \\ + 60 \\ \hline 120 \\ + 120 \\ \hline 240 \\ + 120 \\ \hline 360 \\ + 120 \\ \hline 480 \end{array}$$

$$\begin{array}{r} 40 \\ + 40 \\ \hline 80 \\ + 80 \\ \hline 160 \\ + 80 \\ \hline 240 \\ + 80 \\ \hline 320 \\ + 80 \\ \hline 400 \\ + 80 \\ \hline 480 \end{array}$$

## Error in equation

$$\frac{14 \text{ gal}}{3 \text{ min}} \times \frac{x}{8 \text{ min}} = 14 \times 8 = 112 = 3x = 112 \div 3 = x$$

Rounding error

Computational error

Other

### Remainder error

Bob's shower uses 14 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 8 minutes? Show your work.

$$\frac{14 \text{ gallons}}{3 \text{ minutes}} = \frac{4.2}{1 \text{ minute}}$$

$$3 \overline{) 14} \begin{array}{r} 4 \\ \underline{12} \\ 2 \end{array}$$

$$\begin{array}{r} 4.2 \\ \times 8 \\ \hline 33.6 \end{array}$$

33.6 gallons

## Misinterprets the meaning of quantities

Big Horn Ranch raises 100 horses on 150 acres of pasture. Jefferson Ranch raises 75 horses on 125 acres of pasture. Which ranch has more acres of pasture per horse?.

Big Horn Ranch: Big Horn has 25 more than Jefferson and 25 more acres than Jefferson

Jefferson Ranch: Jefferson has 25 less horses and 25 less acres

Achers

## OGAP Sample Problem Situations and Types (Draft July 2011)

Important: The samples below do not include the full range of problem situations or problem types.

### Ratio - referent explicit:

There are red and blue marbles in a bag. The ratio of red to blue marbles is 1: 2. Sue opened the bag and found 12 red marbles. How many blue marbles are in the bag?

### Ratio - referent implied:

There are red and blue marbles in a bag. The ratio of red to blue marbles is 1: 2. Sue opened the bag and found 12 red marbles. How many marbles are in the bag altogether?

### Rates - Density

Town A	Town B
60 square miles	40 square miles
480 raccoons	380 raccoons

Karl says that Town A has more racoons per square mile.  
Josh says Town B has more racoons per square mile.  
Who is right? Justify your answer.

### Rates - Constant rate of speed

A train is moving at a constant rate of speed. It has traveled 230 miles in 5 hours. At this rate how many miles will the train travel in 9 hours?

### Rate - Buy/consume - Unit Rate

A 20-ounce box of Toasty Oats costs \$2.98. How much does Toasty Oats cost per ounce?

### Rates Comparison

A 20-ounce box of Toasty Oats costs \$2.98. A 16-ounce of Toasty Oats cost \$2.48. Which box of Toasty Oats is a better buy?

### Similarity

The dimensions of 4 rectangles are given below. Which 2 rectangles are similar? Show your work

- a) 2 inches x 3 inches   b) 4 inches x 5 inches   c) 6 inches x 9 inches   d) 6 inches x 10 inches

### Scale Factor

Richard enlarged a photo using a scale factor of 1.8. If the dimensions of the original photo was 5 inches by 7 inches, what are the dimensions of the enlarged photo?

### Missing Value

Bob's shower uses 12 gallons of water in 3 minutes. How many gallons of water does Bob use if he takes a shower in 12 minutes?

### Qualitative

Nick ran exactly the same number of laps around the school track yesterday as he ran today. If it took him less time to run the laps today than yesterday, his running speed today would be:

- a) faster   b) slower   c) exactly the same   d) Not enough information

Explain your answer.

### Non-proportional

Kim and Bob were running equally fast around a track. Kim started first. When she had run 9 laps, Bob had run 3 laps. When Bob had run 15 laps, how many laps had Kim run?