OGAP Multiplicative Framework

(Draft September 2011)

Depending upon the strength of multiplicative reasoning students may move back and forth between using multiplicative, transitional, additive, and non-multiplicative strategies as they interact with different problem structures and problem situations. (Kouba & Franklin, 1995; VMP OGAP, 2006)

PROBLEM SITUATIONS

- Combinations
- Equal groups
- Equal measures
- Equations
- Measure conversions
- Multiplicative change
- Multiplicative comparison
- Patterns
- Properties
- Rate
- Rectangular area
- Volume

PROBLEM STRUCTURES

Types of Items

- Application/context
- Concept/property

Multiplicative Representations

- Equal groups
- Arrays
- Area
- Open area
- Linear

Number of Factors

- Two factors
- More than two factors

Language

- Natural (e.g., every)
- Mathematical (e.g., per)

Complexity of Numbers

Factors

- Single digit
- Multiple digit
- Powers of ten
- Fractions/decimals

Divisors

- Single digit
- Multiple digit
- Powers of ten
- Fractions/decimals

Understanding and Use of Properties

- Associative
- Commutative
- Distributive
- Equality
- Identity
- Multiplicative Inverse

Understanding and Use of Relationships

- Addition-Multiplication
- Doubling and Halving
- Model - Equation
- Model - Model
- Multiples and Factors
- Meanings of remainders

Types of Division

- Partitive
- Quotative

ABOUT OGAP FRAMEWORKS

OGAP Frameworks are based on mathematics education research on how students learn specific mathematics concepts, errors students make, and pre-conceptions or misconceptions that may interfere with learning new concepts or solving related problems. OGAP references are found at http://www.margepetit.com/petit_pdfs/OGAPReferences3.pdf.

There are three major elements to an OGAP Framework that should be considered when analyzing student work or making instructional decisions:

1) problem situations
2) problem structures
3) evidence in student work

This page identifies problem situations and problem structures for multiplication and division problems. Pages 2 and 3 are tools to help teachers classify evidence in student work, including classroom discussions, and make instructional decisions. Page 4 has samples of different problem situations.

For students to become strong multiplicative reasoners they must interact with a range of problem situations and problem structures. The CCSSM specifically identifies problem situations at targeted grades on a progression from equal groups and measures, and area situations at grades 2 and 3 to measurement conversions, volume, and multiplicative change and patterns, and multiplicative comparison problem situations at grades 4 and 5. This progression, among other things, is designed to prepare students to engage in proportional situations using multiplicative reasoning.

Consistent with the CCSSM the OGAP Frameworks on pages 2 and 3 show a progression from the link between repeated addition and multiplication in an equal groups model to the development of efficient and generalizable multiplicative strategies through the open area model, and understanding of place value, properties of operations, and relationships.

As students interact with new concepts, new problem situations, new structures, and more complex problem solving situations they may move back and forth between multiplicative, transitional, additive, and non-multiplicative strategies. This is important evidence to use for instructional decision making. For example, a student may consistently solve equal group problems using a Multiplicative Strategy regardless of the complexity of the numbers, but you may find that the same student adds factors (Non-Multiplicative Strategy) when solving multiplicative change problems.

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OGAP Multiplicative Reasoning Framework - Multiplication (Draft September 2011)

Multiplicative Strategies

Algorithms

Partial Products

16
X 42
12
20
240
400
672

Traditional

21
16
X 42
32
640
672

Distributive Property

16 x 4 = 4 (10 + 6) = 4(10) + 4(6) = 40 + 24 = 64

Associative Property

(8 x 2) x 5 = 8 (2 x 5) = 8 x 10 = 80

Commutative Property

16 x 4 = 4 x 16

Doubling and Halving

16 x 4 = 8 x 8 = 64

Known or Derived Fact

6 x4 = 24

Powers of Ten

5 x 400 = 5 x 4 x 10 x 10

Applies Understanding of Place Values, Properties, and Relationships

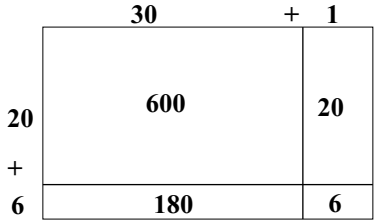
Transitional Multiplicative Strategies

Transitional

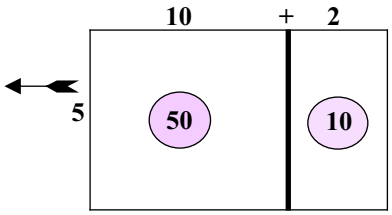
Considers both dimensions of the array and area model.

Open Area Model

26 x 31 = 806

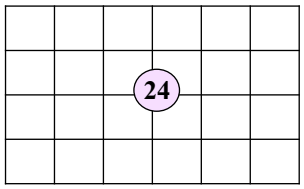


12 x 5 = 60



Area Model

6 x 4 = 24

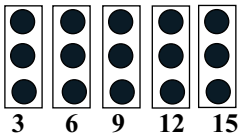


Early Transitional

Skip Counting

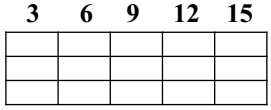
3, 6, 9, 12, 15

Equal groups in an array

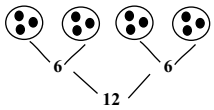


Only considers one dimension of the array and area model.

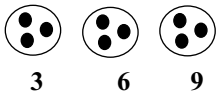
Equal groups in an area model



Building up



3 + 3 + 3 + 3
6 + 6
12



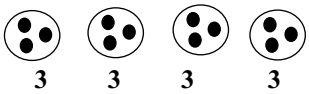
Additive Strategies

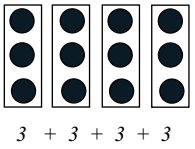
Additive

Repeated addition with or without a model

3 + 3 + 3 + 3 = 12

3 x 4 = 12

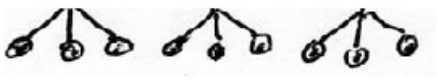




Subitizing in small groups

Early Additive

Modeling - Counting by ones



Modeling - Counting by sub groups 3 x 5 = 15

Unitizes into groups and sub-groups

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Non-Multiplicative Strategies

- Adds or subtracts factors
- Models factors incorrectly
- Uses incorrect operation
- Not enough information
- Guesses

Underlying Issues/Errors

- Misinterprets the remainders
- Units inconsistent or missing
- Calculation error
- Place value error
- Vocabulary error
- Property or relationship error
- Equation error

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OGAP Multiplicative Reasoning Framework - Division (Draft September 2011)

Multiplicative Strategies

Algorithms

Partial Quotients

```
17 | 585
   | 340   x 20
   | 245
   | 170   x 10
   |  75
   |  68   x 4
   |   7   34 r 7
```

```
          4
         10
         20  34 r 7
17 | 585
   | 340
   | 245
   | 170
   |  75
   |  68
   |   7
```

Traditional

```
      34 r 7
17 | 585
   | 51↓
   |  75
   |  68
   |   7
```

Distributive Property

35 ÷ 7 = (21 + 14) ÷ 7 = 3 + 2 = 5

Treats the remainder appropriately given problem situation

Inverse relationship between multiplication and division

35 ÷ 7 = 5 **7 x ? = 35**

Known or Derived Fact

21 ÷ 7 = 3

Transitional Multiplicative Strategies

Transitional

Inefficient partial quotients

```
61 | 756   12 R 24
   | 183   x3
   | 573
   | 183   x3
   | 390
   | 183   x3
   | 207   x3
   | 183
   |  24
```

Trial and error to find a quotient

```
61 | 756
   | 732
   |  24
     12 R 24
```

2 x 61 = 122
20 x 61 = 1220
4 x 61 = 244
8 x 61 = 488
15 x 61 = 915
10 x 61 = 610
12 x 61 = 732

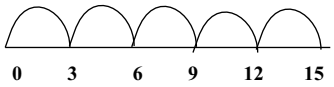
Early Transitional

Models in an array to find missing dimensions 15 ÷ 5 = 3

Skip Counts to find the number of "skips" with and without a model

15 ÷ 3 = 5

3, 6, 9, 12, 15 (5 skips)



Additive Strategies

Additive

Repeated subtraction or adding up to

```
61 | 350   5 r 45
   |  61
   | 289
   |  61
   | 228
   |  61
   | 167
   |  61
   | 106
   |  61
   |  45
```

```
350
-305
 r 45
```

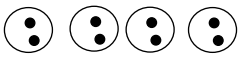
```
 61
+61
122
+61
183
+61
244
+61
305
+61
366
```

Sharing out in equal groups as repeated subtracted or addition

There are 8 cookies to share equally with 4 children. How many cookies does each child get?

8 cookies ÷ 4 children =2 cookies per child

Represents the 4 children with circles and then fills them equally



Twenty-four cookies were put into bags of 4 cookies each. How many bags were filled?

24 cookies ÷ 4 cookies per bag = 6 bags

Pulls out 4 cookies at a time until 24 cookies are used.

Early

Sharing out by ones

Sharing out randomly by subsets

Sharing 4, then 2, then 5 and so on

Applies Understanding of Place Values, Properties, and Relationships

Unitizes into groups and sub-groups

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Adds or subtracts dividends/divisors
Models problem incorrectly
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Not enough information
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Underlying Issues/Errors

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OGAP Sample Problem Situations (Draft September 2011)

Important: The sample problem situations below do not include the full range of each problem situation.

Multiplication Examples	Division Examples How many in each group?(partitive) How many groups? (quotative)
Equal group, measurement conversion, equal measure, and rate problems involve applying a rate. number of groups/measurements/quantities x rate = total number Multiplicative change, multiplicative patterns and multiplicative comparison involve a multiplicative scale factor. original x scale factor = result Area and volume problems involve using dimensions in either an area or volume situation. Combinations involve problems that determine the ways objects can be arranged where order does not matter.	
Equal Group Mark bought 12 boxes of crayons. Each box contained 8 crayons. How many crayons were there all together? 12 boxes x 8 crayons per box = ? crayons	Mark had a box of 64 crayons. He shared the crayons equally with 4 people. How many crayons did each person get? (partitive) 64 crayons ÷ 4 people = ? crayons per person
Equal Measures It takes 14 inches of ribbon to make one bow. How many inches of ribbon will it take to make 7 bows? 7 bows x 14 inches per bow = ? inches	Sam has 15 yards of material. He is making a design that needs 3 yards per design. How many designs can Sam make? (quotative) 15 yards ÷ 3 yards per design = ? designs
Measurement Conversion Tammy is 5 feet tall. How many inches tall is Tammy? 5 feet x 12 inches/foot = ? inches	
Rates Sam works at the grocery store. He is paid $7.00 per hour. He worked 22 hours last week. How much money did Sam earn last week? 22 hours x $7.00/hour = ? dollars	Sam earned $154.00 last week. He worked 22 hours. How much did Sam earn per hour? (partitive) $154.00 ÷ 22 hours = ? dollars per hour
Multiplicative Comparison The students in Mrs. Gilbert's class planted bean and corn seeds. The bean plants grow 3 times faster than the corn plants. When the corn plants measure 2 inches, how tall will the bean plants be? 2 inches x 3 = ? height of bean plants	Bill's garden is 240 square feet. Leslie's garden is 20 square feet. How many times bigger is Bill's garden than Leslie's garden? (quotative) 240 square feet ÷ 20 square feet = ? times bigger
Multiplicative Change/Patterns A 5-inch piece of elastic is stretched 3 times its length. How long is the elastic after it is stretched? (4 times, 5 times, *n* times) 5 inches x 3 = ? (total length)	A piece of elastic stretches to 3 times its length. When fully stretched it is 57 inches long. What is its original length? (partitive) Total length (57 inches) ÷ 3 = ? (original length)
Area Linda's kitchen floor measures 12 feet by 7 feet. How many tiles (1 square foot) are needed to cover the floor? 12 feet x 7 feet = ? (total area in square feet)	Linda's kitchen floor is 150 square feet. The length of one dimension is 10 feet. What is the length of the other dimension of the kitchen floor? 150 square feet ÷ 10 feet = ? (length of other dimension in feet)
Combinations Al bought an ice cream treat (one scoop). He has a choice of sugar cone, waffle cone, or a bowl. There were 5 different flavors of ice cream. How many different combinations can Al choose from? 3 types of cones x 5 flavors = ? combinations	Seth bought some new shirts and pants. He has a total of 12 different outfits. If he bought four pair of pants, how many shirts did Seth buy? 12 outfits ÷ 4 pants = ? shirts
OGAP Equation Example: **6 x 5 = 30** Write a story problem that goes with this equation. OR match a story problem to an equation.	**OGAP Property Example:** Ann knows the answer to 9 x 5. Explain how can she use this information to solve 45 ÷ 9?

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