OGAP Fraction Framework

(Draft Sept 2011)

CONCEPTS

Part whole Equivalence and magnitude Operations

PROBLEM STRUCTURES

FRACTIONS: unit fractions, non-unit fractions, proper fractions, improper fractions mixed numbers, negative fractions, algebraic fractions

Reasoning Strategies

- Number sense
- Unit fraction
- Extended unit fraction
- Modeling
- Benchmark/reference points
- Equivalence
- Common denominators
- Density of fractions

Models

- Area
- Set
- Number Line

To solve problems
To understand concepts
To generalize concepts

Wholes

- Same size
- Different size
- Given part, find whole

Number Lines

- 0 -1
- Negative to positive
- More than 2 units
- Unpartitioned
- Partitioned

Number of Parts in the Whole

relative to the magnitude of the denominator

- Equal
- Multiples
- Factors

In a model or problem situation

Partitioning Strategies

- Algorithmic halving
- Oddness
- Evenness
- Composition

Operations

All Operations

- Estimation
- Efficient algorithm
- Number sense
- Equivalence

Multiplication and Division

- Impact of multiplying and dividing by a fraction.
- Partitive division
- Quotative division

OGAP references are found at http://www.margepetit.com/petit_pdfs/OGAPReferences3.pdf.

ABOUT OGAP FRAMEWORKS

OGAP Frameworks are based on mathematics education research on how students learn specific mathematics concepts, errors students make, and pre-conceptions or misconceptions that may interfere with learning new concepts or solving related problems.

There are three major elements to an OGAP Framework that should be considered when analyzing student work or making instructional decisions:

1) mathematics concepts (part whole, equivalence and magnitude, and operations)
2) problem structures
3) evidence in student work

This page identifies math topics and problem structures for fraction problems. Page 2 is a tool to help teachers classify evidence in student work, including classroom discussions, and make instructional decisions.

Consistent with the CCSSM the OGAP Fraction Framework shows a progression that includes modeling as a means to more generalized reasoning and efficient fractional strategies with the goal to develop understanding of concepts, relationships, and procedural fluency. Ultimately, fluency and understanding will enable students to engage in middle school mathematical topics that assume proficiency with fractions (e.g., proportionality, solving equations with fractional coefficients).

As students interact with new concepts, new structures, and more complex problem solving situations they may move back and forth between fractional, transitional fractional, early fractional, and non-fractional strategies. This is important evidence to use for instructional decision making. For example, a student may consistently find a fractional part of a set by physically partitioning a given set of objects. However, when asked to find 3/4 of 36 students might revert to a non-fractional strategy.

To learn more about the research underpinning the OGAP Fraction Framework read - A Focus on Fractions: Bringing Research to the Classroom (Petit, Laird, and Marsden, 2010).

This is a derivative product of the Vermont Mathematics Partnership Ongoing Assessment Project originally funded by NSF (EHR-0227057) and the US DOE (S366A020002)

OGAP Fraction Framework (draft July 2011)

The examples provided below do NOT represent the full set of possible solutions that represent each level.

As students learn new concepts or interact with new structures or problem situations they may move back and forth across these levels.

Generalizes and Applies to other Mathematical Topics

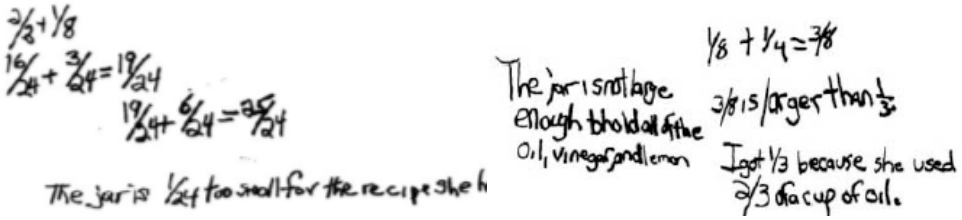
Middle School topics and concepts in which rationale number understandings are applied:

Fractional Strategy

Can accurately locate fractions on a number line of any length, compare and order fractions with a range of strategies, find equivalent fractions, and operate with fractions when solving contextual and non-contextual problems.

- Uses reasoning about relative magnitudes
- Uses benchmark reasoning
- Uses unit fraction reasoning
- Uses extended unit fraction reasoning
- Uses equivalence reasoning
- Uses common denominators
- Uses an efficient algorithm
- Uses "out of equal parts" reasoning

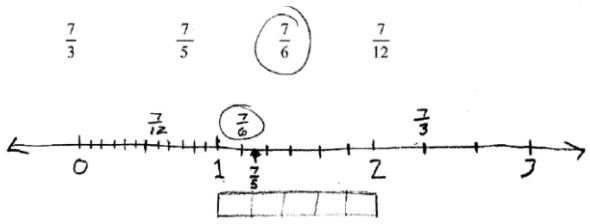
Aunt Sally has a jar that holds one cup of liquid. Her salad dressing recipe calls for 2/3 cup of oil, 1/8 cup of vinegar, and ¼ cup of juice. Is the jar large enough to hold the oil, vinegar, and juice?



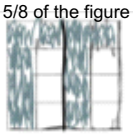
Transitional Fractional Strategy

Effectively generates a model to solve contextual and non- contextual problems

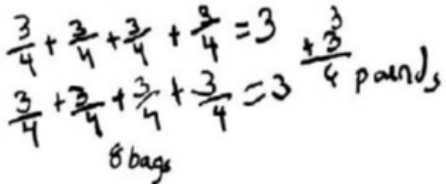
(3) Which fraction is closest to 1? Show your work and reasoning.



Strategy not efficient or generalizable (e.g., "out of parts")

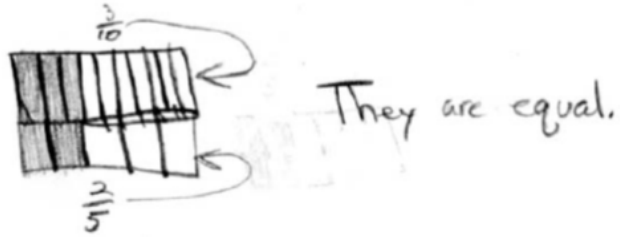


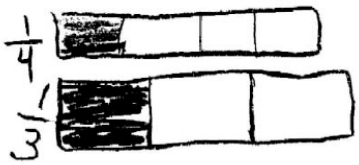
Ashley bought 6 pounds of candy. She put the candy into bags that each hold 3/4 of a pound of candy. How many bags of candy did Ashley fill?



Early Fractional Strategy

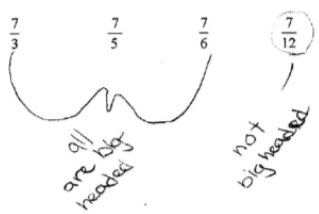
Uses a fractional strategy (like modeling) or an operation appropriate for the situation but the solution includes an error(e.g., calculation, partitioning, size of the whole.)





Non-Fractional Reasoning

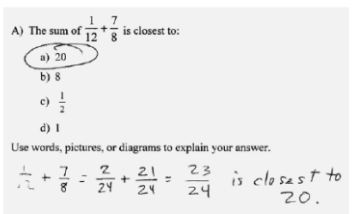
Which fraction is closest to 1?



Stephanie and Paige are discussing the answer to $3\frac{2}{7} \times \frac{5}{7}$.
Stephanie said that the answer is more than $3\frac{2}{7}$.
Paige said the answer is less than $3\frac{2}{7}$.
Who is correct?

if you multiplie any thing it has to be bigger than what you multiplie by. Stephanie is right.

Whole number reasoning, not fractional reasoning.


A) The sum of $\frac{1}{12} + \frac{7}{8}$ is closest to:
a) 20
b) 8
c) $\frac{1}{2}$
d) 1
Use words, pictures, or diagrams to explain your answer.
1/12 + 7/8 = 2/24 + 21/24 = 23/24 is closest to 20.

This is a derivative product of the Vermont Mathematics Partnership Ongoing Assessment Project originally funded by NSF (EHR-0227057) and the US DOE (S366A020002)